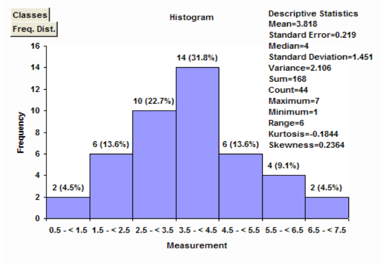
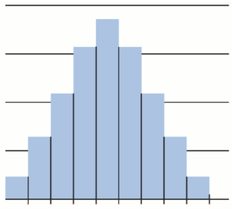
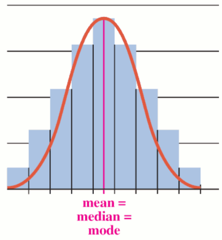
14.4 The Normal Distribution



1

A **Normal Distribution** describes many real life data sets. The data histogram looks like a bell curve.





2

μ (mu) = population mean

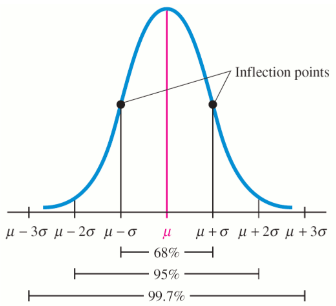
σ (sigma) = population standard deviation

Remarkably, all you need are these two values to know the spread of data about the mean.

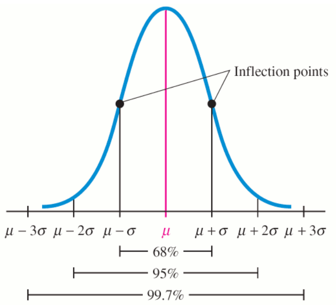
3

μ (mu) = population mean

σ (sigma) = population standard deviation



68% of all data is within 1 std. dev. of the mean.
95% of all data is within 2 std. dev. of the mean.
99.7% of all data is within 3 std. dev. of the mean.



The number of standard deviations a data value is away from the mean is called the z-score.

FORMULA FOR CONVERTING RAW SCORES TO Z-SCORES Assume a normal distribution has a mean of μ and a standard deviation of σ. We use the equation

$$z = \frac{x - \mu}{\sigma}$$

to convert a value x in the nonstandard distribution to a z-score.

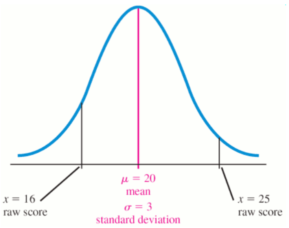
Now every normal distribution can be converted to a **Standard Normal Distribution**, which has $\mu = 0$ and $\sigma = 1$.

6

Example:
Suppose the mean of a normal distribution is 20 and its std dev is 3.

Find the z-score of 25.

Find the z-score of 16.



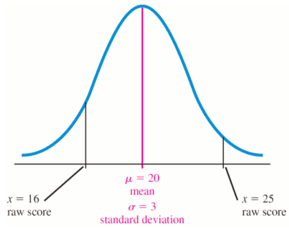
Example:
Suppose the mean of a normal distribution is 20 and its std dev is 3.

Find the z-score of 25.

$$z = \frac{x-\mu}{\sigma} = \frac{25-20}{3} = \frac{5}{3} = 1.67.$$

Find the z-score of 16.

$$z = \frac{x-\mu}{\sigma} = \frac{16-20}{3} = \frac{-4}{3} = -1.33.$$



The area between z-scores gives the percent of data values between them.

Below is a table that gives the area between the mean (μ) and a given z-score.

z	A	z	A	z	A	z	A	z	A	z	A
.00	.000	.56	.212	1.12	.369	1.68	.454	2.24	.488	2.80	.497
.01	.004	.57	.216	1.13	.371	1.69	.455	2.25	.488	2.81	.498
.02	.008	.58	.219	1.14	.373	1.70	.455	2.26	.488	2.82	.498
.03	.012	.59	.222	1.15	.375	1.71	.456	2.27	.488	2.83	.498
.04	.016	.60	.226	1.16	.377	1.72	.457	2.28	.489	2.84	.498
.05	.020	.61	.229	1.17	.379	1.73	.458	2.29	.489	2.85	.498
.06	.024	.62	.232	1.18	.381	1.74	.459	2.30	.489	2.86	.498
.07	.028	.63	.236	1.19	.383	1.75	.460	2.31	.490	2.87	.498
.08	.032	.64	.239	1.20	.385	1.76	.461	2.32	.490	2.88	.498
.09	.036	.65	.242	1.21	.387	1.77	.462	2.33	.490	2.89	.498
.10	.040	.66	.245	1.22	.389	1.78	.463	2.34	.490	2.90	.498

As this gives areas between the middle (mean = μ) and a positive z-score:

Always draw a picture.

For two postive z-scores – subtract areas

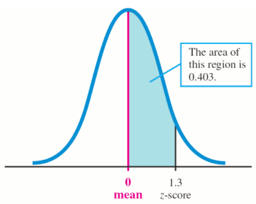
A negative z-score is on the left side of the mean.

Find the percent of data between z=0 and z=1.3

z	A	z	A	z	A	z	A	z	A	z	A
.00	.000	.56	.212	1.12	.369	1.68	.454	2.24	.488	2.80	.497
.01	.004	.57	.216	1.13	.371	1.69	.455	2.25	.488	2.81	.498
.02	.008	.58	.219	1.14	.373	1.70	.455	2.26	.488	2.82	.498
.03	.012	.59	.222	1.15	.375	1.71	.456	2.27	.488	2.83	.498
.04	.016	.60	.226	1.16	.377	1.72	.457	2.28	.489	2.84	.498
.05	.020	.61	.229	1.17	.379	1.73	.458	2.29	.489	2.85	.498
.06	.024	.62	.232	1.18	.381	1.74	.459	2.30	.489	2.86	.498
.07	.028	.63	.236	1.19	.383	1.75	.460	2.31	.490	2.87	.498
.08	.032	.64	.239	1.20	.385	1.76	.461	2.32	.490	2.88	.498
.09	.036	.65	.242	1.21	.387	1.77	.462	2.33	.490	2.89	.498
.10	.040	.66	.245	1.22	.389	1.78	.463	2.34	.490	2.90	.498

Find the percent of data between z=0 and z=1.3

For z = 1.3 A = 0.40



Area between = 0.403 or 40.3%

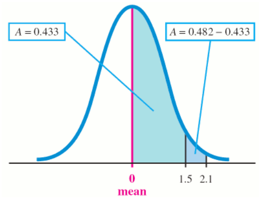
Find the percent of data between z=1.5 and z=2.1

z	A	z	A	z	A	z	A	z	A	z	A
.00	.000	.56	.212	1.12	.369	1.68	.454	2.24	.488	2.80	.497
.01	.004	.57	.216	1.13	.371	1.69	.455	2.25	.488	2.81	.498
.02	.008	.58	.219	1.14	.373	1.70	.455	2.26	.488	2.82	.498
.03	.012	.59	.222	1.15	.375	1.71	.456	2.27	.488	2.83	.498
.04	.016	.60	.226	1.16	.377	1.72	.457	2.28	.489	2.84	.498
.05	.020	.61	.229	1.17	.379	1.73	.458	2.29	.489	2.85	.498
.06	.024	.62	.232	1.18	.381	1.74	.459	2.30	.489	2.86	.498
.07	.028	.63	.236	1.19	.383	1.75	.460	2.31	.490	2.87	.498
.08	.032	.64	.239	1.20	.385	1.76	.461	2.32	.490	2.88	.498
.09	.036	.65	.242	1.21	.387	1.77	.462	2.33	.490	2.89	.498
.10	.040	.66	.245	1.22	.389	1.78	.463	2.34	.490	2.90	.498

Find the percent of data between z=1.5 and z=2.1

For z = 1.5 A = 0.482

For z = 2.1 A = 0.433



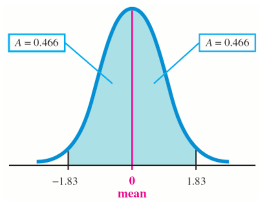
Area between = 0.482 – 0.433 = 0.051 or 5.1%

Find the percent of data between z=0 and z=1.3

z	A	z	A	z	A	z	A	z	A	z	A
.00	.000	.56	.212	1.12	.369	1.68	.454	2.24	.488	2.80	.497
.01	.004	.57	.216	1.13	.371	1.69	.455	2.25	.488	2.81	.498
.02	.008	.58	.219	1.14	.373	1.70	.455	2.26	.488	2.82	.498
.03	.012	.59	.222	1.15	.375	1.71	.456	2.27	.488	2.83	.498
.04	.016	.60	.226	1.16	.377	1.72	.457	2.28	.489	2.84	.498
.05	.020	.61	.229	1.17	.379	1.73	.458	2.29	.489	2.85	.498
.06	.024	.62	.232	1.18	.381	1.74	.459	2.30	.489	2.86	.498
.07	.028	.63	.236	1.19	.383	1.75	.460	2.31	.490	2.87	.498
.08	.032	.64	.239	1.20	.385	1.76	.461	2.32	.490	2.88	.498
.09	.036	.65	.242	1.21	.387	1.77	.462	2.33	.490	2.89	.498
.10	.040	.66	.245	1.22	.389	1.78	.463	2.34	.490	2.90	.498

Find the percent of data between z=-1.83 and z=0

For z = 1.83 A = 0.466
So z = -1.83 A = 0.466
on the other side.



Area between = 0.466 or 46.6%

We can also go backwards.

Find a z-score so that 37.1% of the data is between the mean and it.

z	A	z	A	z	A	z	A	z	A	z	A
.00	.000	.56	.212	1.12	.369	1.68	.454	2.24	.488	2.80	.497
.01	.004	.57	.216	1.13	.371	1.69	.455	2.25	.488	2.81	.498
.02	.008	.58	.219	1.14	.373	1.70	.455	2.26	.488	2.82	.498
.03	.012	.59	.222	1.15	.375	1.71	.456	2.27	.488	2.83	.498
.04	.016	.60	.226	1.16	.377	1.72	.457	2.28	.489	2.84	.498
.05	.020	.61	.229	1.17	.379	1.73	.458	2.29	.489	2.85	.498
.06	.024	.62	.232	1.18	.381	1.74	.459	2.30	.489	2.86	.498
.07	.028	.63	.236	1.19	.383	1.75	.460	2.31	.490	2.87	.498
.08	.032	.64	.239	1.20	.385	1.76	.461	2.32	.490	2.88	.498
.09	.036	.65	.242	1.21	.387	1.77	.462	2.33	.490	2.89	.498
.10	.040	.66	.245	1.22	.389	1.78	.463	2.34	.490	2.90	.498

We can also go backwards.

Find a z-score so that 37.1% of the data is between the mean and it.

z = 1.13

If μ = 100 and σ = 20, find the raw data x.

We can also go backwards.

Find a z-score so that 37.1% of the data is between the mean and it.

$z = 1.13$

If $\mu = 100$ and $\sigma = 20$, find the raw data x.

$z = (x - \mu)/\sigma$
$1.13 = (x - 100)/20$
$22.6 = x - 100$
$x = 122.6$

Overall idea:

Raw data x ↔ z-score ↔ areas

use: $z = \frac{x - \mu}{\sigma}$ Table